

Presentation Assignment 3

1. Suppose that $\text{Card}(A) = \text{Card}(C)$ and $\text{Card}(B) = \text{Card}(D)$. Show that $\text{Card}(A \times B) = \text{Card}(C \times D)$. [Hint: There is a bijective map $f: A \rightarrow C$ and a bijective map $g: B \rightarrow D$. Use these functions to construct a bijective function $h: A \times B \rightarrow C \times D$.]
2. True or false? An *arbitrary* union of countable sets must be countable. If true, prove it. If false, give a counterexample.
3. Let A be a countable subset of $(0, 1)$ and suppose that we can arrange A into the following sequence base 4:

0.1333...

0.0311...

0.1031...

0.2213...

- (a) Find the number $0.a_{11}a_{22}a_{33}a_{44}\dots$ obtained by Cantor's diagonalization argument using the algorithm

$$a_m = \begin{cases} 2 & \text{if the } n, n \text{ entry in matrix is } 1 \\ 0 & \text{otherwise} \end{cases}$$

- (b) Does this diagonalization algorithm produce a number distinct from all those numbers listed in the matrix?